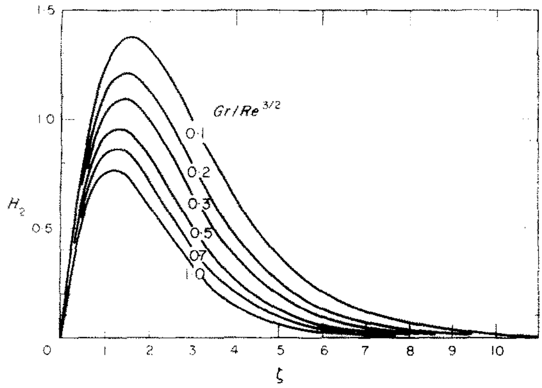

FIG. 5. Temperature profile H_2.

and horizontal temperature gradient, Figs. 3 and 4, do not change significantly. Thus the effect of buoyancy is observed to be more pronounced on the secondary motion than on the rotation of the fluid. Likewise, the change in frictional resistance to the radial flow is larger than the change in heat transfer. The heat transfer due to T_2 is into the disc in the case $T_0 = T_\infty$. But this is only a small fraction of the total heat transfer at large distances from the axis. The component T_2 is driven mainly by the radial conduction of heat due to T_1 when $T_0 = T_\infty$. This energy can only go into the wall. The profile of T_2, Fig. 5, gradually collapses as $Gr/Re^{3/2}$ increases.

At $Gr/Re^{3/2} = 1$, the velocities induced by buoyancy are of the same order as those due to rotation. Hence, to get results further, the equations should be recast into a form based on quantities characterizing natural convection flow on the lines followed by Rotem and Claassen [4].

REFERENCES

1. T. VON KÁRMÁN, Available as NACA-TM No. 1092 (1966).
2. I. B. DUNCAN, *J. Fluid Mech.* **24**, 417 (1966).
3. K. H. LUK, K. MILLSAPS and K. POHLHAUSEN, IV International Heat Transfer Conference, Paris 1970, Vol. IV, paper No. NC4.2.
4. Z. ROTEM and L. CLAASSEN, IV International Heat Transfer Conference, Paris 1970, Vol. IV, paper no. NC4. 3.
5. A. RALSTON and H. WILF, *Mathematical Methods for Digital Computers*. Wiley, New York (1965).
6. S. K. SREENIVASAN, M. Tech. Thesis, Dept. of Mech. Engng, Indian Institute of Technology, Kanpur, India (May 1969).
7. G. K. BATCHELOR, *Q.J. Mech. Appl. Math.* **4**, 29 (1951).
8. R. L. YOUNG, *Trans. Am. Soc. Mech. Engrs* **78**, 1163 (1956).
9. E. C. COBB and O. A. SAUNDERS, *Proc. Camb. Phil. Soc.* **30**, 365 (1956).
10. P. D. RICHARDSON and O. A. SAUNDERS, *J. Mech. Engng Sci.* **5**, 336 (1963).

Int. J. Heat Mass Transfer. Vol. 16, pp. 1492–1496. Pergamon Press 1973. Printed in Great Britain

VIEW FUNCTION IN GENERALIZED CURVILINEAR COORDINATES FOR SPECULAR REFLECTION OF RADIATION FROM A CURVED SURFACE

DONALD G. BURKHARD and DAVID L. SHEALY

Department of Physics and Astronomy, University of Georgia, Athens, Georgia 30601, U.S.A.

(*Received* 1 *August* 1972 *and in revised form* 20 *November* 1972)

NOMENCLATURE

dS_0, dS_1, dS_2, element of area of emitter, reflector, and receiver;

$(x_0, y_0, z_0), (x, y, z), (X, Y, Z)$, coordinates of dS_0, dS_1 and dS_2, respectively;

r, distance from emitter, dS_0, to reflector, dS_1;

r', distance from reflector, dS_1, to receiver, dS_2;

ρ, reflectivity of reflector;

B, brightness of source dS_0;
$\boldsymbol{n}_0, \boldsymbol{n}_1, \boldsymbol{n}_2$, unit normal of emitter, reflector and receiver, respectively;
$\boldsymbol{i}$, unit vector in the direction of radiation from emitter to reflector;
$\boldsymbol{i}'$, unit vector in the direction of the radiation from reflector to receiver;
$d\varepsilon_{dS_0-dS_2}$, energy flux density for radiation emitted by dS_0, specularly reflected by dS_1, and received by dS_2;
$dF_{dS_0-dS_2}$, specular view function which represents that portion of radiant flux leaving source dS_0, specularly reflected by dS_1, and incident upon dS_2.

1. INTRODUCTION

In reference [1] a technique is described for calculating the flux density, that is, the energy per unit area per unit time over an arbitrary receiver surface when the incident radiation is specularly reflected from an arbitrary reflecting surface. The surface equation of both the receiver and reflector surface are expressed in the explicit cartesian form, $z = z(x, y)$. In this paper the problem is formulated in general curvilinear coordinates. An expression is derived for the view function for radiation leaving an element of area dS_0, specularly reflected by dS_1 and incident upon dS_2. These results are used to calculate the heat flux emitted from a fin at one end of a cylinder, reflected by the cylinder and received by a fin at the other end. Results may be compared with the same problem treated by the image method of [2].

2. GENERAL FORMALISM

The energy flux per unit receiver area dS_2 for radiation which is emitted by dS_0, intercepted by dS_1 and then specularly reflected to dS_2 is given by the flux flow equation [1]

$$d\varepsilon_{dS_0-dS_2} = \frac{\rho B \boldsymbol{n}_0 . \boldsymbol{i}\boldsymbol{n}_1 . \boldsymbol{i} dS_0}{\pi r^2 |dS_2/dS_1|}. \tag{1}$$

Equations for the reflector surface and the receiver surface in parametric form are

$$\boldsymbol{x} = x(u, v)\boldsymbol{I} + y(u, v)\boldsymbol{J} + z(u, v)\boldsymbol{K} \tag{2a}$$

$$\boldsymbol{X} = X(U, V)\boldsymbol{I} + Y(U, V)\boldsymbol{J} + Z(U, V)\boldsymbol{K} \tag{2b}$$

where u, v and U, V are curvilinear coordinates [3]. The equation of the reflected ray is

$$\begin{aligned} F_1(u, v; U, V) &= i'_z(u, v)[X(U, V) - x(u, v)] \\ &\quad -(u, v)[Z(U, V) - z(u, v)] = 0 \\ F_2(u, v; U, V) &= i'_z(u, v)[Y(U, V) - y(u, v)] \\ &\quad -(u, v)[Z(U, V) - z(u, v)] = 0 \end{aligned} \tag{3}$$

where (i'_x, i'_y, i'_z) specify the direction of reflected ray which satisfies the law of reflection and are the components of

$$\boldsymbol{i}' = \boldsymbol{i} - 2\boldsymbol{n}_1(\boldsymbol{i} . \boldsymbol{n}_1).$$

Analogous to [1] the ratio dS_2/dS_1 appearing in (1) can be written as

$$\frac{dS_2}{dS_1} = \frac{|(\partial \boldsymbol{X}/\partial U) \times (\partial \boldsymbol{X}/\partial V)|}{|(\partial \boldsymbol{x}/\partial u) \times (\partial \boldsymbol{x}/\partial v)|} \frac{\partial(U, V)}{\partial(u, v)} \tag{4}$$

where the functional relationship between u, v and U, V required to evaluate the jacobian $\partial(U, V)/\partial(u, v) \equiv (\partial U/\partial u)(\partial V/\partial v) - (\partial U/\partial v)(\partial V/\partial u)$ is given by (3). From the quotient property of jacobians [3]

$$\frac{\partial(U, V)}{\partial(u, v)} = \frac{\partial(F_1, F_2)/\partial(u, v)}{\partial(F_1, F_2)/\partial(U, V)}. \tag{5}$$

The jacobians $\partial(F_1, F_2)/\partial(u, v)$ and $\partial(F_1, F_2)/\partial(U, V)$ can be evaluated directly from (3) by using the chain rule for partial differentiation, for example,

$$\frac{\partial F_1}{\partial u} = -i'_z\frac{\partial x}{\partial u} + i'_x\frac{\partial z}{\partial u} - (Z - z)\frac{\partial i'_x}{\partial u} + (X - x)\frac{\partial i'_z}{\partial u}.$$

One obtains the results

$$\begin{aligned} \partial(F_1, F_2)/\partial(u, v) &= i'_z\{I_0 + r'I_1 + (r')^2 I_2\} \\ \partial(F_1, F_2)/\partial(U, V) &= i'_z\{\boldsymbol{i}' . (\partial \boldsymbol{X}/\partial U) \times (\partial \boldsymbol{X}/\partial V)\} \end{aligned} \tag{6}$$

where

$$I_0 = \boldsymbol{i}' . (\partial \boldsymbol{x}/\partial u) \times (\partial \boldsymbol{x}/\partial v)$$

$$I_1 = \boldsymbol{i}' . [(\partial \boldsymbol{x}/\partial u) \times (\partial \boldsymbol{i}'/\partial v) + (\partial \boldsymbol{i}'/\partial u) \times (\partial \boldsymbol{x}/\partial v)]$$

$$I_2 = \boldsymbol{i}' . (\partial \boldsymbol{i}'/\partial u) \times (\partial \boldsymbol{i}'/\partial v)$$

$$r' = (Z - z)/i'_z.$$

Combining (4), (5) and (6) with (1) gives an explicit expression for $d\varepsilon_{dS_0-dS_2}$.

Since the view function for specular reflection, $dF_{dS_0-dS_2}$, represents that portion of the radiant flux leaving an element of area dS_0, specularly reflected by dS_1, and then incident upon dS_2 one can write a general expression for the specular view function as

$$\begin{aligned} dF_{dS_0-dS_2} &= \frac{[(d\varepsilon_{dS_0-dS_2})\, dS_2]}{\int_{S_0} B\, dS_0} \\ &= \frac{\rho \boldsymbol{n}_0 . \boldsymbol{i}\boldsymbol{n}_1 . \boldsymbol{i}\boldsymbol{n}_2 . \boldsymbol{i}' |(\partial \boldsymbol{x}/\partial u) \times (\partial \boldsymbol{x}/\partial v)| dS_0\, dS_2}{S_0 \pi r^2 |I_0 + r'I_1 + (r')^2 I_2|}. \end{aligned} \tag{7}$$

The brightness B is regarded as constant over S_0. The terms I_0, I_1, I_2 express the role of the curvature of the reflector surface in the final expression for $d\varepsilon_{dS_0-dS_2}$ or $dF_{dS_0-dS_2}$.

It is possible to express (7) in terms of the intrinsic geometry of the reflector surface, that is, the gaussian, the mean, and the normal curvature. When this is done and the curvature is allowed to go to zero, that is, when the element of reflector surface is degenerated to a flat element of surface, (7) becomes

$$dF_{dS_0-dS_2} = -\frac{\rho \boldsymbol{n}_0 \cdot \boldsymbol{i} dS_0 dS_2}{\pi S_0 (r+r')^2}. \quad (8)$$

Equation (8) is the formula given by [2] for the specular view function but we see that all curvature terms are missing. Thus view function of [2] does not represent the proper generalization to curved surfaces.

using (8). Because of the axial symmetry the energy flux per unit area emitted by the right fin and received by the left fin is independent of the azimuthal angle α_2 of the receiver coordinates. Thus, we can fix dS_2 at $\alpha_2 = 0$. The element of receiver area dS_2 is defined by a point $P_2 = (X, Y, Z) = (0, p, 0)$ where p is the radial distance from the origin of the left fin. The emitter point and reflector point are similarly defined by the points $p_0 = (q \sin \alpha_0, q \cos \alpha_0, L)$ and $P_1 = (x, y, z) = (a \sin \omega, a \cos \omega, z)$, respectively, where q and α_0 are the radial and azimuthal coordinates of the emitting point on the right fin, and z and ω are the axial and azimuthal coordinates of the reflecting point on the cylinder wall. In

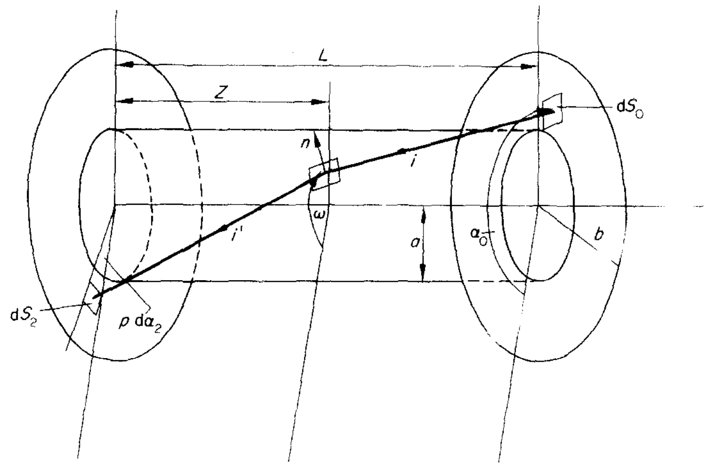

FIG. 1. Double finned cylinder.

terms of these coordinates one can write the following expressions:

The energy received per unit area of receiver surface, $F_{S_0-dS_2}$, after being specularly reflected by the reflecting surface expressed as a fraction of the total energy emitted by the source is obtained by dividing (7) by dS_2 and integrating over S_0. Similarly, the fraction of the total energy $F_{S_0-S_2}$ emitted by the source surface, specularly reflected, and received by the receiver surface is obtained by integrating (7) over S_0 and S_2.

3. EXAMPLE

As an example of the use of the preceding formulas we shall calculate the fraction of the energy emitted by a fin S_0, specularly reflected by a cylinder, S_1, and received by a second fin, S_2 (Fig. 1). This will be shown as a function of the receiving fin radial coordinate (flux contours). The total specular view factor, $F_{S_0-S_2}$, for the receiving fin will also be calculated. The latter calculation is performed in [2] using (8).

1. Normal on reflecting surface:

$$\boldsymbol{n}_1 = \sin \omega \boldsymbol{I} + \cos \omega \boldsymbol{J}. \quad (9a)$$

2. Unit vector along direction of incident radiation form P_0 to P_1:

$$\boldsymbol{i} = [(a \sin \omega - q \sin \alpha_0)\boldsymbol{I} + (a \cos \omega - q \cos \alpha_0)\boldsymbol{J} + (z - L)\boldsymbol{K}]/r \quad (9b)$$

where $r^2 = a^2 + q^2 + (L - z)^2 - 2aq \cos(\alpha_0 - \omega)$.

3. Unit vector along direction of reflected radiation form P_1 to P_2:

$$\boldsymbol{i}' = \{[q \sin(2\omega - \alpha_0) - a \sin \omega]\boldsymbol{I} + [q \cos(2\omega - \alpha_0) - a \cos \omega]\boldsymbol{J} + (Z - L)\boldsymbol{K}\}/r. \quad (9c)$$

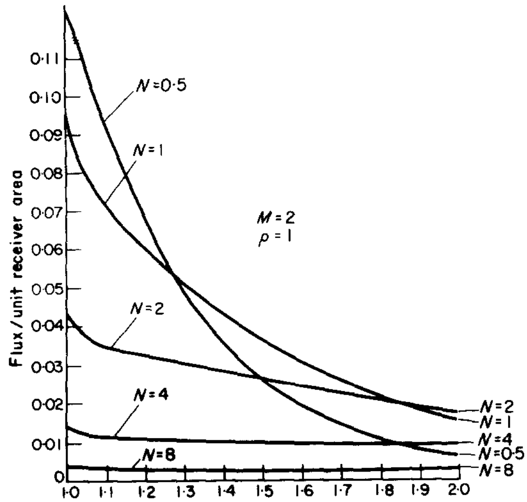

FIG. 2. Fraction of energy emitted by right fin, specularly reflected from cylinder wall and intercepted per unit area of receiving fin as a function of dimensionless radial coordinate p/a for $M = b/a = 2$.

Combining (9c) with (3) one has the following equations for the reflected ray:

$$X = a \sin \omega + z[q \sin (2\omega - \alpha_0) - a \sin \omega]/(L - z)$$

$$Y = a \cos \omega + z[q \cos (2\omega - \alpha_0) - a \cos \omega]/(L - z). \tag{10}$$

Evaluating the functions I_0, I_1, I_2 from (6) with z, ω as the variables u, v and assuming that the reflectivity of the cylinder wall is unity one obtains the following expression for fraction of the total energy flux per unit area on receiver surface, S_2 in terms of the dimensionless quantities $\zeta = p/a, \eta = q/a, \beta = z/a, M = b/a$, and $N = L/a$;

$$\frac{dF_{S_0-dS_2}}{dS_2} = \frac{1}{\pi^2(M^2-1)}\int_1^M d\eta \int_{-\cos^{-1}(1/\zeta)-\cos^{-1}(1/\eta)}^{\cos^{-1}(1/\zeta)+\cos^{-1}(1/\eta)} d\alpha_0$$

$$\times \frac{\eta(\eta \cos(\omega-\alpha_0)-1)(\beta-N)^4}{[1+\eta^2+(N-\beta)^2-2\eta\cos(\omega-\alpha_0)]^2 \,|[\eta N(4\beta-N) \times \cos(\omega-\alpha_0)-2\beta N(1+\eta^2)+N^2|} \tag{11}$$

where the limits of α_0 are such that $\boldsymbol{n}.\boldsymbol{i} = 0$. Before (11) can be integrated, one must eliminate β and ω. This can be done by solving (10) for $z(= \beta a)$ and ω

$$\beta = N(\zeta - \cos\omega)/[\zeta + \eta\cos(2\omega - \alpha_0) - 2\cos\omega]$$

$$\sin(2\omega - \alpha_0) = \eta^{-1}\sin\omega + \zeta^{-1}\sin(\omega - \alpha_0). \tag{12}$$

The results for $(dF_{S_0-dS_2})/dS_2$ as a function of ζ for $N = 0·5$, 1, 2, 4, 8 and $M = 2$ are presented in Fig. 2.

Integrating (11) over $dS_2(\equiv p\,dp\,d\alpha_2 = \zeta\,d\zeta\,d\alpha_2)$ gives the fraction of the total energy $F_{S_0-S_2}$ emitted by the source, specularly reflected and received by S_2. The results for $F_{S_0-S_2}$

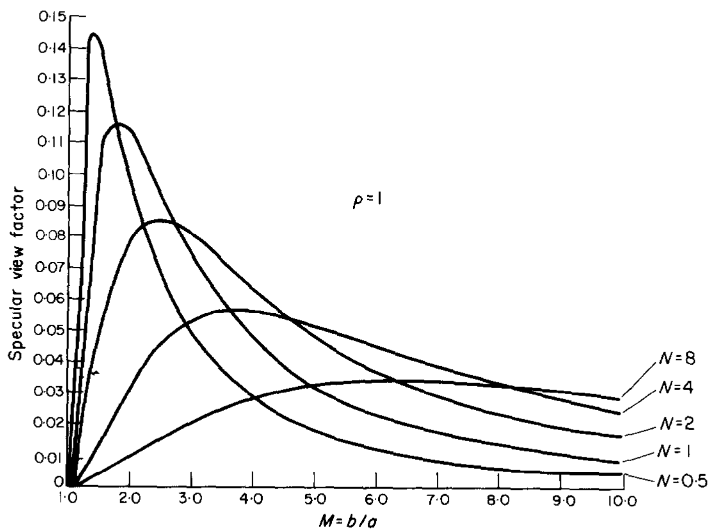

FIG. 3. Fraction of energy emitted by right fin, specularly reflected by cylinder wall, and received by left fin.

as a function of M for five different values of N are presented in Fig. 3.

For comparison, the fraction of the total energy emitted by S_0 and directly incident upon S_2, without reflection, is shown in Fig. 4.

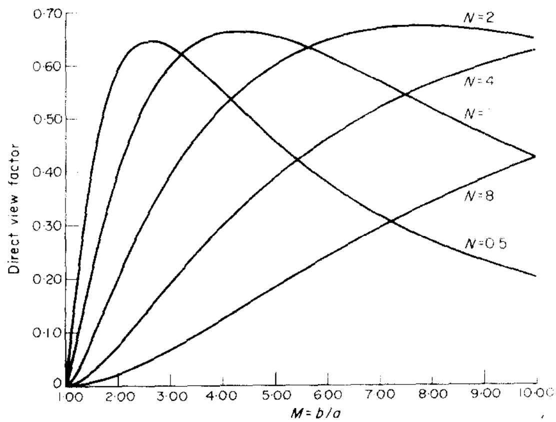

FIG. 4. Fraction of energy emitted by right fin and received directly by left fin without reflection.

REFERENCES

1. D. G. BURKHARD, D. L. SHEALY and R. U. SEXL, Specular reflection of radiation from an arbitrary reflection surface to an arbitrary receiver surface, *Int. J. Heat Mass Transfer* **16**, 271–280 (1973).
2. J. A. PLAMONDON and T. E. HORTON, On the determination of the view function to the images of a surface in a nonlinear specular reflector, *Int. J. Heat Mass Transfer* **10**, 665–679 (1967).
3. M. R. SPIEGEL, *Advanced Calculus*. Schaum's Outline Series, McGraw-Hill, New York (1963).

Int. J. Heat Mass Transfer. Vol. 16, pp. 1496–1500. Pergamon Press 1973. Printed in Great Britain

MULTIVALUED RELATIONS BETWEEN SURFACE CONDUCTION AND SURFACE TEMPERATURE IN A SATURATED POROUS MEDIA WITH PHASE CHANGE

SAMUEL SCHWEITZER
School of Engineering, Tel-Aviv University, Ramat-Aviv, Israel

(*Received* 1 *November* 1972 *and in revised form* 8 *December* 1972)

NOMENCLATURE

C_p, specific heat;
h, enthalpy;
h_{fg}, latent heat of vaporization;
k, thermal conductivity;
K, permeability;
L, length of porous section;
$\dot{m}$, mass flow rate per unit area $= \rho v$;
p, pressure;
Pe, Péclét number $= mC_pL/k = RePrL/d$;
q, heat flux per unit area;
S, interface position;
T, temperature;
x, distance coordinate.

Greek symbols
Δ, conduction–convection difference;
θ, dimensionless temperature;
μ, viscosity;
ν, kinematic viscosity $= \mu/\rho$;
ρ, convection–conduction ratio.

Dimensionless quantities
C, $\equiv C_{pL}/C_{pv}$;
F, $\equiv \dot{m}_L/\dot{m}_i$;
H, $\equiv h_{fg}(T^*)/[h_L(T^*) - h_L(T_R)]$;
R, $\equiv v_v/v_L$;
δ, $\equiv L\Delta/kv_{eff}(T^* - T_R)$;
κ, $\equiv k_{L_{eff}}/k_{v_{eff}}$;
ψ, $\equiv d\hat{q}_0/dS$.